

TIME EVOLUTION OF $K^0 - \bar{K}^0$ SYSTEM IN SPECTRAL FORMULATION

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Abstract

We reanalyse the time evolution of the $K^0 - \bar{K}^0$ system in the language of certain spectral function whose Fourier transforms give the time dependent survival and transition amplitudes. The reanalysis turned out to be necessary in view of the astonishing theorem by Khalfin on the possibility of vacuum regeneration of K_S and K_L . The main reason for this unexpected behaviour is the non-orthogonality of $|K_S\rangle$ and $|K_L\rangle$. As a result of this theorem new contributions to the well known oscillatory terms will enter the time dependent transition probabilities. These new terms are not associated with small/large time behaviour of the amplitudes and therefore their magnitude is a priori unknown. Approximating the spectral functions by an one-pole ansatz Khalfin estimated the new effect in transition probabilities to be 4×10^{-4} . Whereas we agree with Khalfin on the general existence of vacuum regeneration of K_S and K_L we disagree on the size of the effect. A careful analysis of the one-pole approximation reveals that the effect is eleven orders of magnitude smaller than Khalfin's estimate and, in principle, its exact determination lies outside the scope of the one-pole ansatz. The present paper gives also insight into the limitation of the validity of one-pole approximation, not only for small/large time scales, but also for intermediate times where new effects, albeit small, are possible. It will be shown that the same validity restrictions apply to the known formulae of Weisskopf-Wigner approximation as well.

1 Introduction

The present paper reconsiders an old subject of quantum mechanical time evolution of the $K^0 - \bar{K}^0$ system. Instead of applying the well known Weisskopf-Wigner (WW) approach [1] to the $K^0 - \bar{K}^0$ system [2] we examine the time evolution in the spectral formalism which is often employed for unstable quantum mechanical systems [3]. In this formulation the Fourier transform of a spectral density function gives the time dependent transitions and survival amplitudes. The reasons to pick up once again the old subject of time development are twofold. Since the WW approach is an approximation it is rather useful to have yet another, different formalism which either confirms the WW results (within a certain accuracy) or is capable of displaying new (howsoever small) effects. Due to some peculiarities of the $K^0 - \bar{K}^0$ system one might indeed suspect that the limitations of the applicability of the WW approximations are, in principle, different as compared with other quantum mechanical systems (see below). In view of the planned high precision experiments in this system it is then not unreasonable to reconsider this subject. Secondly, the more specific reason for this reanalysis is a result by Khalfin on the possibility of vacuum regeneration of K_S and K_L [4], [5], [6]. The latter would induce new terms in the time development formulae which according to Khalfin are not completely negligible. In this paper we investigate this possibility by using a more refined analysis than Khalfin's.

The $K^0 - \bar{K}^0$ complex is one of the most important test grounds of basic symmetry properties of nature, like CP- and eventually CPT-(non)conservation [7], [8], [9], [10]. It has also been realized that the $K^0 - \bar{K}^0$ system can be used as a sensitive probe of one of the fundamental aspect of the theory of nature, namely Quantum Mechanics [9], [10]. This and the fact that the $K^0 - \bar{K}^0$ system is till now the only system to show experimental evidence of CP-violation makes it clear why this specific subject has always played an almost outstanding role in particle physics. Since the discovery of CP-violation in 1964 [11] an enormous number of papers has been devoted to this subject, but even today it is an alive area and both, the experiment and the theory, try to infer more information towards a better understanding of CP-violation. We still lack an experimental confirmation of direct CP-violation (predicted by the Standard Model) in contrast to the experimentally established fact of CP-violation through mixing. Indeed, the two different measurements of $\Re(\epsilon'_K/\epsilon_K)$ [12], [13], one of which [13] is compatible with zero, are inconclusive in this respect and further measurements are eagerly awaited (the ratio $\Im(\epsilon'_K/\epsilon_K)$ is at present also consistent with zero [14]). From the theoretical side the basic framework to calculate ϵ_K and ϵ'_K in the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ Standard Model and its various extensions is well understood [15]. However, the extraction of exact numbers predicted from the theory is still hampered by hadronic uncertainties, like the bag factor, by poorly known Kobayashi-Maskawa element V_{ub} and the CP-violating phase δ as well as by the still not very precisely known top mass value. Recently further progress has been made by realizing the importance of gluonic corrections [16], [17] due to the large top

mass and that the latter makes it necessary to include the contributions of electroweak Penguins [17], [18].

It is a well appreciated fact that many new models beyond the Standard Model have to pass the test of $K^0 - \bar{K}^0$ physics putting sometimes severe restrictions on the model parameters [19]. So, for instance, the *general* two Higgs doublet model which without any further restrictions would predict flavour changing neutral currents (FCNC) making the transitions $K^0 \leftrightarrow \bar{K}^0$ possible at tree level is usually supplemented by a discrete Z_2 symmetry to avoid FCNC [20]. Further examples are the Supersymmetric version of the Standard Model where off-diagonal gluino-squark-quark coupling gives rise to a new CP-violating source in the strong sector of the theory which then appears in a gluino mediated box graph [21]. An even more ‘exotic’ example of a $K^0 \leftrightarrow \bar{K}^0$ transition is provided by the R-parity broken SUSY model where, in principle, this transition can happen through an exchange of a sneutrino [22]. Last but not the least, rare Kaon decays can also put severe limits on new speculative physics [23] and the connection can go as far as to the, by now excluded, ‘fifth force’ [24]. All this shows the high sensitivity of the $K^0 - \bar{K}^0$ system. It is then not a surprise that one can use the $K^0 - \bar{K}^0$ physics as a testing ground of even more speculative assumptions, notably CPT-violation and violation of Quantum Mechanics. Eventough both these topics were almost sacrosanct, recent advances in string theory [25] and formal developments in Quantum Mechanics pioneered by Bell [26] made it more plausible that violation of both might actually occur in nature. As far as CPT-violation is concerned we do not expect the latter to happen in the context of local, causal and Lorentz-invariant Quantum Field Theories (QFT). Indeed, the famous CPT-theorem [27] assures us that with the three aforementioned conditions CPT is conserved on very general grounds. To circumvent this theorem one has to drop one of the three underlying conditions. Probably the least painful way would be to drop the requirement of locality. Such QFT’s have been discussed in the literature, but the status of their consistency is in doubt. String theories offer possibility to attack this problem: displaying in some sense a non-local interaction, but being consistent on the other hand. Motivated by the peculiar feature of Hawking radiation of a Black Hole [28] which allows pure states to evolve into mixed states, in contradiction with quantum-mechanical results, a density matrix formalism for the $K^0 - \bar{K}^0$ system [29] based on string theory has been developed [25] which indeed violates CPT. This would be CPT-violation through violation of Quantum Mechanics (see also [30]). If such a prediction comes true it could also be considered as a experimental hint towards string theories (it is interesting to observe the broad span which connects the physics of a Black Hole with the physics e.g. at DaΦne). Quite independent how a possible CPT-violation arises the test of the ‘last discrete space-time symmetry’ which seems unbroken till now is important (for the status of CPT-violation from experiment see [31]).

Also independent of any specific theory a precision experiment of Quantum Theory in the $K^0 - \bar{K}^0$ system is desirable. Doubts about validity of Quantum Theory

in general date back to the birth of Quantum Mechanics highlighted by arguments like the EPR-paradox [32] and speculations about hidden variable theories [33]. A general set up of a local realistic models versus Quantum Mechanics has been reanalysed by Bell [26] providing us with the tool of the known Bell-inequalities which arise in the context of a possible hidden variable theory. Experiments with spin correlations show that this inequality is violated [34] and hence QM confirmed, at least in this case. In ref.[35] a version of Bell-inequalities has been derived whose examinations revealed that these inequalities are not violated by QM predictions for any choice of the parameters. However, recently a proposal has been made to test QM versus a local theory at the Φ -factory with the help of Bell-inequalities by using $K_L - K_S$ regeneration in *matter* [36]. A suggested test of quantum mechanical superposition principle [37] can also be counted in the realm of general tests of QM. For yet different possibilities and developments we refer the reader to [38], [39].

It is worth stressing that many ongoing and suggested tests, as well as their refutals, of CP-, T-, [40], CPT-symmetry and QM have directly to do with the time evolution of the system. This brings us back to the quantum mechanical time development which is indeed, beside the theoretical determination of the system parameters ϵ_K and ϵ'_K , the second pillar of the $K^0 - \bar{K}^0$ system and which is much less model dependent than the latter. Keeping in mind that any possible violation of CPT and QM is forced to be rather small it is quite important to examine the nature of new effects the time development might hide beyond the WW approximation (the WW approach is an approximation, though a rather good one). To understand why deviations from WW are expected let us recall a well founded theorem which confirms deviations from the exponential decay law $\exp(-\Gamma t)$ for very small (the region of ‘quantum Zeno’ effect [41]) and very large times [42]. It is also known that the exponential decay law can be derived consistently up to terms of order Γ/M [43] which in the case of interest is

$$\frac{\Gamma_X}{m_X} \sim 10^{-15}, \quad X = K_S, K_L \quad (1.1)$$

That the situation in the $K^0 - \bar{K}^0$ system might be different can be seen from the following reasoning. First, due to mixing the mass difference $m_L - m_S$ will enter the transition probabilities like $|\langle K^0 | \bar{K}^0(t) \rangle|^2$ etc. We then find, in addition to (1.1), other dimensionless quantities like

$$\begin{aligned} \frac{\Gamma_S}{m_L - m_S} \sim \mathcal{O}(1), \quad \frac{m_L - m_S}{m_L} \sim 10^{-15} \\ \frac{\Gamma_L}{m_L - m_S} \sim 10^{-3} \end{aligned} \quad (1.2)$$

Of course no new effects will be present which go hand in hand with the first ratio. It is the third dimensionless ratio in (1.2) which is intriguing and which is small enough to be dropped in the first approximation, but on the other hand not small enough to be neglected completely.

The second reason why the $K^0 - \bar{K}^0$ system differs from a ‘normal’ unstable quantum mechanical system is that the K_S and K_L , defined as usual, are not orthogonal to each other due to the presence of CP-violation in the mixing. This peculiar property causes sometimes problems like e.g. EPR-like paradox [44] and gives rise to questions for the anti-particles of K_S and K_L . For a more detailed discussion on this issue we refer the reader to the papers [45]. Based on this non-orthogonality Khalfin has proved, in the formalism of spectral functions ρ_S and ρ_L (suitable also otherwise for any unstable quantum mechanical system) that the *vacuum* (in contrast to similar phenomena in *matter*) regeneration probability of $K_S \leftrightarrow K_L$ is *non-zero* unless there is no CP-violation through mixing in the $K^0 - \bar{K}^0$ system [5], [6], [46]. To estimate this effect he uses a reasonable one-pole approximation for ρ_S and ρ_L and finds then indeed *new* terms in the transition probability $|\langle K^0 | \bar{K}^0(t) \rangle|^2$ etc. which are of the order of $\Gamma_L / (m_L - m_S)$. We agree with Khalfin on a general existence of such an effect of vacuum regeneration of K_S and K_L once the K_S and K_L are defined in the usual manner. But we disagree on the numerical estimate of this effect. It will be shown below that a consistent treatment of the spectral formalism in general and the one-pole approximation in specific yields a quite different picture as far as the size of this ‘new’ effect is concerned. Indeed Khalfin does not use all the information available in the formalism which, in our opinion, leads to the wrong estimate. In detail the following will be shown below

- (i) Taking into account all available information on the spectral functions ρ_S and ρ_L we investigate the consistency of the one-pole approximation and find that it is valid up to terms of order Γ_X / m_X , $(m_L - m_S) / m_L$. It will be argued that such corrections do arise *not only* for very large and very small time scales.
- (ii) Through this consistency check we can determine all parameters of the one-pole approximation needed for the time evolution formulae (again up to accuracy of Γ_X / m_X , $(m_L - m_S) / m_L$) in terms of known quantities.
- (iii) This makes it possible to derive time evolution formulae like $|\langle K^0 | \bar{K}^0(t) \rangle|^2$ etc. in the spectral formalism and with the one-pole ansatz (in the accuracy mentioned above) without any further assumptions. The result of a lengthy calculation is that all formulae agree with the corresponding expressions derived within the WW approach.
- (iv) In consequence this result shows explicitly that the vacuum regeneration probability must be of the order Γ_X / m_X , $(m_L - m_S) / m_L$. This, however, does not mean that that such an effect is associated with small/large time behaviour of the amplitudes.

This work was inspired by a talk given by Khalfin in the Second DaΦne Meeting. After the main bulk of the work has been finished the author of the present

paper became aware of a paper by Chiu and Sudershan [46] who treat the same subject. These authors use the solvable Friedrichs-Lee model to show that, in general, Khalfin's conclusions on the vacuum regeneration are indeed correct. However, they disagree with Khalfin's numerical estimate. The present paper arrives at the same conclusion as [46] with the difference that no specific model is needed. It will be shown below that one can arrive at these conclusions by a careful analysis of the one-pole ansatz.

The paper is organized as follows. In section 2 we collect all essential and quite general formulae for the time development. In section 3 we present two of Khalfin's results. Section 3 investigates the one-pole ansatz and its consistency. In section 4 all the forgoing results will be gathered to derive the time evolution of the system. In section 5 we present our conclusions.

2 Basic Formulae

Out of the Weisskopf-Wigner approximation we will essentially need only the part which has to do with the eigenvectors of the effective, non-hermitian Hamiltonian which is the result of two approximations made in the Schrödinger equation [47], [48]. This part defines the K_S and K_L states in the usual way

$$|K_S\rangle = p|K^0\rangle + q|\bar{K}^0\rangle, \quad |K_L\rangle = p|K^0\rangle - q|\bar{K}^0\rangle \quad (2.1)$$

$$\langle K_S|K_S\rangle = \langle K_L|K_L\rangle = |p|^2 + |q|^2 = 1 \quad (2.2)$$

$$\langle K_S|K_L\rangle = \langle K_L|K_S\rangle = |p|^2 - |q|^2 \neq 0 \quad (2.3)$$

The equality $\langle K_S|K_L\rangle = \langle K_L|K_S\rangle$ in eq.(2.3) is imposed by CPT-invariance which we will assume to hold throughout the paper. The presence of CP-violation in the mixing is reflected by $|p|^2 - |q|^2 \neq 0$ which enforces the states K_S and K_L to be non-orthogonal to each other. Another often used parametrization of the mixing parameters is given by

$$p = \frac{1 + \epsilon_K}{\sqrt{2(1 + |\epsilon_K|^2)}}, \quad q = \frac{1 - \epsilon_K}{\sqrt{2(1 + |\epsilon_K|^2)}} \quad (2.4)$$

which makes contact with the ϵ_K parameter mentioned in the introduction. Since the CP-violation in the $K^0 - \bar{K}^0$ system (or equivalently the non-orthogonality of K_S and K_L) will play an important role we define for the sake of a short notation

$$\Delta_K \equiv |p|^2 - |q|^2 \quad (2.5)$$

Let us also note here that although eqs.(2.1)-(2.3) come out naturally in the context of WW-approximation, independent of this approximation, assuming $K^0 \leftrightarrow \bar{K}^0$ mixing, the presence of CP-violation in the mixing and implementing therein CPT-contraints there is not much choice left other than to postulate eqs.(2.1)-(2.3) for the K_S and

K_L states, up to possible contributions from continuum states which we neglect (for a different point of view where in the context of a generalized quantum mechanical vector space K_S and K_L are orthogonal see [45] and references therein). Hence eqs.(2.1)-(2.3) have a much broader applicability than the part of WW approximation which determines the time dependence of transition and survival amplitudes.

Given a full, hermitian Hamiltonian H according to general principles of Quantum Mechanics the time evolution for K^0 and \bar{K}^0 can be summarized as follows

$$\begin{aligned} P_{K_\alpha K_\beta}(t) &= \langle K_\alpha | e^{-iHt} | K_\beta \rangle = \langle K_\alpha | K_\beta(t) \rangle \\ |K_\alpha(t)\rangle &= e^{-iHt} |K_\alpha\rangle \\ K_\alpha &= K^0, \bar{K}^0, \end{aligned} \quad (2.6)$$

Due to the non-orthogonality of K_S and K_L there is a subtle difference between the treatment of the time evolution of K^0 , \bar{K}^0 and K_S , K_L . For the former the $P_{K_\alpha K_\beta}(t)$ are expansion coefficients in

$$\begin{aligned} |K^0(t)\rangle &= P_{K^0 K^0}(t) |K^0\rangle + P_{\bar{K}^0 K^0}(t) |\bar{K}^0\rangle \\ |\bar{K}^0(t)\rangle &= P_{\bar{K}^0 \bar{K}^0}(t) |\bar{K}^0\rangle + P_{K^0 \bar{K}^0}(t) |K^0\rangle \end{aligned} \quad (2.7)$$

which according to the orthogonality of K^0 and \bar{K}^0 and in agreement with the first equation in (2.6) are identical to $\langle K_\alpha | K_\beta(t) \rangle$ for $K_\alpha = K^0, \bar{K}^0$. Since the quantum mechanical principle $|A(t)\rangle = \exp(-iHt)|A\rangle$ is valid for any state $|A\rangle$ we can use eqs.(2.1)-(2.3) and eq.(2.6) to derive the following time dependence of K_S and K_L

$$\begin{aligned} |K_S(t)\rangle &= p \left[P_{K^0 K^0}(t) |K^0\rangle + P_{\bar{K}^0 K^0}(t) |\bar{K}^0\rangle \right] + q \left[P_{\bar{K}^0 \bar{K}^0}(t) |\bar{K}^0\rangle + P_{K^0 \bar{K}^0}(t) |K^0\rangle \right] \\ |K_L(t)\rangle &= p \left[P_{K^0 K^0}(t) |K^0\rangle + P_{\bar{K}^0 K^0}(t) |\bar{K}^0\rangle \right] - q \left[P_{\bar{K}^0 \bar{K}^0}(t) |\bar{K}^0\rangle + P_{K^0 \bar{K}^0}(t) |K^0\rangle \right] \end{aligned} \quad (2.8)$$

Note that in this section we are keeping all formulae as general as possible, in accordance with the general principles of Quantum Mechanics. In analogy to eq.(2.7) and again in full generality we can also define expansion coefficients $P_{K_S K_S}(t)$, $P_{K_L K_L}(t)$, $P_{K_L K_S}(t)$ and $P_{K_S K_L}(t)$ through

$$\begin{aligned} |K_S(t)\rangle &= P_{K_S K_S}(t) |K_S\rangle + P_{K_L K_S}(t) |K_L\rangle \\ |K_L(t)\rangle &= P_{K_L K_L}(t) |K_L\rangle + P_{K_S K_L}(t) |K_S\rangle \end{aligned} \quad (2.9)$$

Clearly the time dependent functions $P_{K_S K_L}(t)$ and $P_{K_L K_S}(t)$, absent in the WW approximation, would be, unless identical to zero, responsible for vacuum regeneration of $K_S \leftrightarrow K_L$. Using already the following CPT-constraint (being at same time a quite model-independent test for CPT conservation [6])

$$P_{K^0 K^0}(t) = P_{\bar{K}^0 \bar{K}^0}(t) \quad (2.10)$$

the $P_{K_S K_S}(t)$ etc can be easily obtained from (2.8) by using the inverse transformation of eq.(2.1). The result is

$$\begin{aligned} P_{K_S K_S}(t) - P_{K_L K_L}(t) &= \frac{q}{p} P_{K^0 \bar{K}^0}(t) + \frac{p}{q} P_{\bar{K}^0 K^0}(t) \\ P_{K_S K_S}(t) + P_{K_L K_L}(t) &= P_{K^0 K^0}(t) + P_{\bar{K}^0 \bar{K}^0}(t) = 2P_{K^0 K^0}(t) \\ P_{K_L K_S}(t) = -P_{K_S K_L}(t) &= \frac{1}{2} \left\{ \frac{q}{p} P_{K^0 \bar{K}^0}(t) - \frac{p}{q} P_{\bar{K}^0 K^0}(t) \right\} \end{aligned} \quad (2.11)$$

Trivially eqs.(2.9) imply a relation between the expansion coefficients $P_{K_S K_S}(t)$ etc and the corresponding matrix elements $\langle K_S | K_S(t) \rangle$ etc.

$$\begin{aligned} \langle K_S | K_S(t) \rangle &= P_{K_S K_S}(t) + P_{K_L K_S}(t) \Delta_K \\ \langle K_S | K_L(t) \rangle &= P_{K_S K_S}(t) \Delta_K + P_{K_L K_S}(t) \\ \langle K_L | K_L(t) \rangle &= P_{K_L K_L}(t) - P_{K_L K_S}(t) \Delta_K \\ \langle K_L | K_S(t) \rangle &= P_{K_L K_L}(t) \Delta_K - P_{K_L K_S}(t) \end{aligned} \quad (2.12)$$

This explicitly displays the above mentioned difference between the K^0 , \bar{K}^0 and the K_S , K_L cases. The matrix element e.g. $\langle K_L | K_S(t) \rangle$ is not equal to the corresponding coefficient $P_{K_L K_S}(t)$. Only if *both*, $P_{K_L K_S}(t) = -P_{K_S K_L}(t) = 0$ and $\Delta_K = 0$, are imposed is this equality guaranteed. Hence this property, $\langle K_L | K_S(t) \rangle \neq P_{K_L K_S}(t)$, has nothing to do with the generality of our formulae, but in general with the fact that $\Delta_K \neq 0$.

Let us now come to the main point of the paper. The question which will be addressed in the next sections is whether

$$P_{K_L K_S}(t) = -P_{K_S K_L}(t) = 0 \text{ or } \neq 0 \quad (2.13)$$

As discussed in the introduction Khalfin has proved [5], [6] (confirmed in [46]) that indeed the second possibility must be true unless there is no CP-violation in the mixing, i.e. $\Delta_K = 0$. We will describe Khalfin's result in the next section. Before doing so let us state explicitly that in the WW approximation we have $P_{K_L K_S}(t) = -P_{K_S K_L}(t) = 0$ and that the K_S and K_L have the simple time evolution

$$\begin{aligned} P_{K_S K_S}(t)|_{WW} &= e^{-im_S t} e^{-\frac{1}{2}\Gamma_S t} \\ P_{K_L K_L}(t)|_{WW} &= e^{-im_L t} e^{-\frac{1}{2}\Gamma_L t} \end{aligned} \quad (2.14)$$

as would have been expected for physical, unstable particle states (which do not mix). As discussed above even in the WW approximation we have

$$\langle K_L | K_S(t) \rangle|_{WW} \neq 0, \quad \langle K_S | K_L(t) \rangle|_{WW} \neq 0 \quad (2.15)$$

It is also useful to derive two further relations which will be the cornerstones of the discussion in the next sections. The first one follows immediately from eq.(2.12) and reads

$$\langle K_S | K_L(t) \rangle + \langle K_L | K_S(t) \rangle = \Delta_K [\langle K_L | K_L(t) \rangle + \langle K_S | K_S(t) \rangle] \quad (2.16)$$

This expression will lead in the next section to a relation between the spectral density functions ρ_S and ρ_L . This in turn will yield a couple of consistency equation when the spectral functions are approximated by a one-pole ansatz. To obtain the second relation we have to essentially invert the formulae (2.11) and express the $P_{K^0 K^0}(t)$ etc matrix elements through the expansion coefficients $P_{K_S K_S}(t)$ etc.

$$P_{K^0 \bar{K}^0}(t) = \frac{p}{q} \left\{ \frac{1}{2} [P_{K_S K_S}(t) - P_{K_L K_L}(t)] + P_{K_L K_S}(t) \right\} \quad (2.17)$$

$$P_{\bar{K}^0 K^0}(t) = \frac{q}{p} \left\{ \frac{1}{2} [P_{K_S K_S}(t) - P_{K_L K_L}(t)] - P_{K_L K_S}(t) \right\} \quad (2.18)$$

$$P_{K^0 K^0}(t) = P_{\bar{K}^0 \bar{K}^0}(t) = \frac{1}{2} [P_{K_S K_S}(t) + P_{K_L K_L}(t)] \quad (2.19)$$

Setting therein $P_{K_L K_S}(t) = 0$ we get

$$\frac{P_{K^0 \bar{K}^0}(t)}{P_{\bar{K}^0 K^0}(t)} = \frac{p^2}{q^2} = \text{const} \quad (2.20)$$

This last equation will, when rewritten in the spectral language, lead to $\Delta_K = 0$. Hence the conclusion of Khalfin that $P_{K_S K_L}(t) \neq 0$.

3 Spectral Formulation

What we called spectral formalism for unstable quantum mechanical systems is based on two observations. The first one is simply the completeness of the eigenvectors $|q\rangle$ of a hermitian quantum mechanical Hamiltonian. We can then write an unstable state $|\lambda, t\rangle$ (which is never an eigenstate of the Hamiltonian) as

$$|\lambda, t\rangle = \sum_q |q, t\rangle \langle q|\lambda\rangle \quad (3.1)$$

The second observation is the reasonable assumption that the unstable state has only projections on continuum states in which it decays. Denoting from now on the continuous eigenvalue of a Hamiltonian by m we can write the survival amplitude $A(t)$ (or, as in case of $K^0 \leftrightarrow \bar{K}^0$ oscillations, transition amplitude) as

$$A(t) = \int_{\text{Spec}(H)} dm e^{-imt} \rho(m) \quad (3.2)$$

where the integration extends over the whole spectrum of the Hamiltonian and $\rho(m)$ is

$$\rho(m) = |\langle m|\lambda\rangle|^2 \quad (3.3)$$

Of course the spectrum of any sensible Hamiltonian should be bounded from below. The ground state (vacuum) can be then normalized to have zero energy eigenvalue.

The integration range in (3.2) is in this case from 0 to ∞ . Despite this cut-off in the integral (3.2) imposed on us by physical requirements we stress that $A(t)$ and $\rho(m)$ are still Fourier-transforms of each other. This is guaranteed by the Dirichlet-Jordan (see e.g. [49]) conditions for Fourier integrals which under certain conditions (which we assume here to be fulfilled) allow us to introduce a finite number of discontinuities in the Fourier integrals. At the discontinuous points the result of the Fourier transform will be $1/2[f(x+0) + f(x-0)]$ and simply $f(x)$ otherwise. * With the following Breit-Wigner ansatz (see [43])

$$\rho_{BW}(m) = \frac{\Gamma}{2\pi} \frac{1}{(m - m_0)^2 + \frac{\Gamma^2}{4}} \quad (3.4)$$

we obtain then for the survival amplitude

$$A_{BW}(t) = \int_{-\infty}^{\infty} dm e^{-imt} \rho_{BW}(m) = e^{-im_0 t} e^{-\frac{1}{2}\Gamma t}, \quad t \geq 0 \quad (3.5)$$

which gives for the survival probability the well known exponential decay law, $P_{BW}(t) = |A(t)|^2 = \exp(-\Gamma t)$. Despite of what has been said about the integration range above we have integrated in (3.5) over $(-\infty, \infty)$ for reasons which will be evident in section 5. There it will become apparent that taking the integral from $-\infty$ to ∞ is in some sense equivalent to neglecting terms of order Γ/M (where M is the mass). The existence of a ground state in $\text{Spec}(H)$ introduces non-exponential corrections (and non-oscillatory terms in $P_{K^0 K^0}(t)$ etc.) which, however, using the simple ansatz (3.4) cannot be trusted [43]. We will discuss this ansatz further in section 4.

We can now apply the above formalism to the case of K_S and K_L by introducing a hermitian Hamiltonian with, as before, continuous spectrum of the decay products which we label by indices α, β etc.

$$H|\phi_\alpha\rangle = m|\phi_\alpha\rangle, \quad \langle\phi_\beta(m')|\phi_\alpha(m)\rangle = \delta_{\alpha\beta}\delta(m' - m) \quad (3.6)$$

The unstable states K_S and K_L are then written in accordance with (3.1) as superpositions of the eigenkets.

$$\begin{aligned} |K_S\rangle &= \int_0^\infty dm \sum_\alpha \rho_{S,\alpha}(m) |\phi_\alpha\rangle \\ |K_L\rangle &= \int_0^\infty dm \sum_\beta \rho_{L,\beta}(m) |\phi_\beta\rangle \end{aligned} \quad (3.7)$$

Note that this can be done for any unstable state. Therefore, strictly speaking, equations (3.7) are as such not the definitions of $|K_S\rangle$ and $|K_L\rangle$. The latter are still defined as linear superposition of $|K^0\rangle$ and $|\bar{K}^0\rangle$ in eq.(2.1).

*The other above mentioned conditions are (a) piecewise continuity (except at isolated points), (b) bounded total variation and (c) $\int_{-\infty}^{\infty} dt |A(t)| < \infty$. It is then sufficient to define $\rho(m) \neq 0$ for $m \geq 0$ and $\rho(m) = 0$ for $m < 0$. The absolute integrability is obvious.

In what follows we convert the general formulae of section 2 into the language of spectral functions $\rho(m)$. To do so we first write down the matrix elements from eq.(2.12). Using (3.6) and (3.7) they are given by

$$\begin{aligned}
\langle K_S | K_S(t) \rangle &= \int_0^\infty dm \sum_\alpha |\rho_{S,\alpha}(m)|^2 e^{-imt} \\
\langle K_L | K_L(t) \rangle &= \int_0^\infty dm \sum_\beta |\rho_{L,\beta}(m)|^2 e^{-imt} \\
\langle K_S | K_L(t) \rangle &= \int_0^\infty dm \sum_\gamma \rho_{S,\gamma}^*(m) \rho_{L,\gamma}(m) e^{-imt} \\
\langle K_L | K_S(t) \rangle &= \int_0^\infty dm \sum_\sigma \rho_{L,\sigma}^*(m) \rho_{S,\sigma}(m) e^{-imt}
\end{aligned} \tag{3.8}$$

Eq.(2.16) can be then recast in the following form

$$\begin{aligned}
&\int_0^\infty dm \sum_\alpha \left[\rho_{L,\alpha}^*(m) \rho_{S,\alpha}(m) + \rho_{S,\alpha}^*(m) \rho_{L,\alpha}(m) \right] e^{-imt} \\
&= \Delta_K \int_0^\infty dm \sum_\beta \left[|\rho_{L,\beta}(m)|^2 + |\rho_{L,\beta}(m)|^2 \right] e^{-imt}
\end{aligned} \tag{3.9}$$

Taking the inverse Fourier transform of (3.9) we arrive at

$$\sum_\alpha \left[\rho_{L,\alpha}^*(m) \rho_{S,\alpha}(m) + \rho_{S,\alpha}^*(m) \rho_{L,\alpha}(m) \right] = \Delta_K \sum_\beta \left[|\rho_{L,\beta}(m)|^2 + |\rho_{L,\beta}(m)|^2 \right] \tag{3.10}$$

which is valid for $m \in (0, \infty)$. This equation is one of Khalfin's main results [4] and will play an important role in the subsequent discussion. It tells us that the spectral functions $\rho_{S,\alpha}$ and $\rho_{L,\alpha}$ are inter-related with each other and any reasonable ansatz which approximates these functions should be such that eq.(3.10) is true at least to certain accuracy. Indeed an ansatz for $\rho_{S,\alpha}$ and $\rho_{L,\alpha}$ similar to (3.4) does not fulfill this requirements in full generality and in section 4 we address this question in more detail. Note also that since eq.(3.10) is an equation in the variable m we might expect that given a certain ansatz for the spectral functions we get more than one consistency equations from it.

To obtain the second main result of Khalfin [5], [6] it is necessary to derive corresponding spectral expression for $P_{K^0 K^0}(t)$ etc. From (2.12), (2.17)-(2.19), (2.16) (alternatively (3.10)) and (3.8) we see that

$$\begin{aligned}
P_{K^0 K^0}(t) &= P_{\bar{K}^0 \bar{K}^0}(t) = \int_0^\infty dm \rho_{K^0 K^0}(m) e^{-imt} \\
&= \frac{1}{2} \int_0^\infty dm \sum_\alpha \left\{ |\rho_{S,\alpha}(m)|^2 + |\rho_{L,\alpha}(m)|^2 \right\} e^{-imt} \\
P_{K^0 \bar{K}^0}(t) &= \int_0^\infty dm \rho_{K^0 \bar{K}^0}(m) e^{-imt} = \frac{1}{4p^*q} \int_0^\infty dm \sum_\beta \left\{ |\rho_{S,\beta}(m)|^2 - |\rho_{L,\beta}(m)|^2 \right\}
\end{aligned} \tag{3.11}$$

$$- \rho_{S,\beta}^*(m) \rho_{L,\beta}(m) + \rho_{L,\beta}^*(m) \rho_{S,\beta}(m) \Big\} e^{-imt} \quad (3.12)$$

$$P_{\bar{K}^0 K^0}(t) = \int_0^\infty dm \rho_{\bar{K}^0 K^0}(m) e^{-imt} = \frac{1}{4pq^*} \int_0^\infty dm \sum_\sigma \left\{ |\rho_{S,\sigma}(m)|^2 - |\rho_{L,\sigma}(m)|^2 \right. \\ \left. + \rho_{S,\sigma}^*(m) \rho_{L,\sigma}(m) - \rho_{L,\sigma}^*(m) \rho_{S,\sigma}(m) \right\} e^{-imt} \quad (3.13)$$

Here $\rho_{\bar{K}^0 K^0}(m)$ etc. are simply defined by the right hand sides of the corresponding equations. As done at the end of the forgoing section if we now set $P_{K_L K_S}(t) = -P_{K_S K_L}(t) = 0$ we obtain the spectral version of (2.20)

$$\int_0^\infty dm \rho_{K^0 \bar{K}^0}(m) e^{-imt} = \frac{p^2}{q^2} \int_0^\infty dm \rho_{\bar{K}^0 K^0}(m) e^{-imt} \quad (3.14)$$

By observing from (3.12) and (3.13) that $\rho_{K^0 \bar{K}^0} = \rho_{\bar{K}^0 K^0}^*$ and taking again the inverse Fourier transform in (3.14) we get

$$\frac{p^2}{q^2} = \frac{\rho_{K^0 \bar{K}^0}}{\rho_{\bar{K}^0 K^0}^*} \quad (3.15)$$

This, however, immediately leads to

$$\Delta_K = |p|^2 - |q|^2 = 0 \quad (3.16)$$

Hence Khalfin's second result states that putting $P_{K_L K_S}(t) = -P_{K_S K_L}(t)$ to zero invariably implies that on consistency grounds there can be no CP-violation in the mixing provided the K_S and K_L states are defined as in eqs.(2.1). In other words since we know that CP-violation exists in the mixing of $K^0 - \bar{K}^0$ we have to allow for *vacuum* regeneration of K_S and K_L . Note that this conclusion does not depend on a particular choice of $\rho_{S,\alpha}$ and $\rho_{L,\alpha}$. This is quite an astounding and unexpected result which, using a completely different approach, has also been recently confirmed [46]. It is not easy to give an interpretation of this result. Either we accept (2.1) and the fact that the non-orthogonality of K_S and K_L makes this system different from any other (recall our discussion of this peculiarity in the introduction) known system (except for similar system with the same properties like $B^0 - \bar{B}^0$ or $D^0 - \bar{D}^0$) or we can suspect that (2.1) is not the complete relation [45]. The confirmation of the above result by Chiu and Sudershan [46] shows that this result is indeed reliable. We emphasize this because of its rather 'exotic' implications.

It is also worthwhile noting that the above result has been derived within the context of standard Quantum Mechanics and that CPT-symmetry has been implemented. Suggested tests of CPT and Quantum Mechanics based on terms which are in general forbidden by CPT or QM are then not affected by this result provided the chosen observables assume zero values in the limit of CPT conservation or in

the context of QM. Any other tests which rely on standard WW expressions might, however, be affected. This is true regardless of the size of this new effect and more importantly this effect has nothing to do with deviations of the exponential decay law for very small and very large time. The latter will become manifest in the formulae for time evolution in section 5.

It is nevertheless mandatory to try to estimate the size of this effect. A first step in this direction will be to make an ansatz for the spectral functions $\rho_{S,\alpha}$ and $\rho_{L,\alpha}$ and to check the consistency of this ansatz. Therefore we collect below all available expressions which can shed some light on the spectral functions. From (2.1)-(2.3) we get

$$\int_0^\infty dm \sum_\alpha |\rho_{S,\alpha}(m)|^2 = \int_0^\infty dm \sum_\beta |\rho_{L,\beta}(m)|^2 = 1 \quad (3.17)$$

$$\int_0^\infty dm \sum_\sigma \Im m \left(\rho_{S,\sigma}^*(m) \rho_{L,\sigma}(m) \right) = 0 \quad (3.18)$$

$$\int_0^\infty dm \sum_\gamma \Re \left(\rho_{S,\gamma}^*(m) \rho_{L,\gamma}(m) \right) = \Delta_K \quad (3.19)$$

Eqs.(3.18) and (3.19) follow from (2.3) and the fact that Δ_K is real. Together with (3.10) these equations is all the information on spectral functions $\rho_{S,\alpha}$ and $\rho_{L,\alpha}$ which is given to us. Any ansatz for the spectral functions has to respect these relations, up to a reasonable accuracy. We already mention that Khalfin in his estimate (see also [46] where Khalfin's results and estimate are discussed) used essentially only eq.(3.17). We also point out that once eq.(3.10) and (3.17) are assumed to hold eq.(3.19) follows.

4 One-Pole Approximation and its Consistency

We have seen that the Breit-Wigner ansatz led to the well known exponential decay law (up to corrections induced by the existence of the ground state). It is therefore reasonable to assume a similar form for the $\rho_{S,\alpha}$ and $\rho_{L,\alpha}$. More specifically we write

$$\begin{aligned} \rho_{S,\beta}(m) &= \sqrt{\frac{\Gamma_S}{2\pi}} \frac{A_{S,\beta}(K_S \rightarrow \beta)}{m - m_S + i\frac{\Gamma_S}{2}} \\ \rho_{L,\beta}(m) &= \sqrt{\frac{\Gamma_L}{2\pi}} \frac{A_{L,\beta}(K_L \rightarrow \beta)}{m - m_L + i\frac{\Gamma_L}{2}} \end{aligned} \quad (4.1)$$

where $A_{S,\alpha}$ and $A_{L,\alpha}$ are decay amplitudes. It is convenient to make the following definitions

$$\gamma_S \equiv \frac{\Gamma_S}{2}, \quad \gamma_L \equiv \frac{\Gamma_L}{2}, \quad \Delta m \equiv m_S - m_L \quad (4.2)$$

$$S \equiv \sum_{\alpha} |A_{S,\alpha}|^2, \quad L \equiv \sum_{\alpha} |A_{L,\alpha}|^2 \quad (4.3)$$

$$R \equiv \sum_{\sigma} \Re \left(A_{S,\sigma}^* A_{L,\sigma} \right), \quad I \equiv \sum_{\sigma} \Im m \left(A_{S,\sigma}^* A_{L,\sigma} \right) \quad (4.4)$$

The quantities (4.3) and (4.4) are the only apriori unknown variables which, with the spectral functions given by (4.1), will enter e.g. equations like (3.11)-(3.13). As already mentioned at the end of the last section we have to insert (4.1) into the expressions (3.10) and (3.17)-(3.19) to examine the consistency of the one-pole approximation (4.1).

We start with eq.(3.17) where the integral can be easily performed. The result is

$$S = 1 + \frac{\gamma_s}{\pi m_s} + \mathcal{O}((\gamma_s/m_s)^2), \quad L = 1 + \frac{\gamma_L}{\pi m_L} + \mathcal{O}((\gamma_L/m_L)^2) \quad (4.5)$$

For reasons explained below we will keep, up to a certain point, terms of order Γ_x/m_x . Since (3.10) contains the variable m plugging the one-pole approximation (4.1) in (3.10) we get a polynomial in the variable m of degree two which should be identically zero. Therefore coefficient of each power in m should be also zero. Instead of one equation we have three consistency equations.

$$\begin{aligned} m^2 \left[2\sqrt{\gamma_s \gamma_L} \cdot R - \Delta_K (\gamma_s \cdot S + \gamma_L \cdot L) \right] &= 0 \\ m \left[-2\sqrt{\gamma_s \gamma_L} (m_L + m_s) \cdot R - 2\sqrt{\gamma_s \gamma_L} (\gamma_s - \gamma_L) \cdot I + 2\Delta_K (\gamma_s m_L \cdot S + \gamma_L m_s \cdot L) \right] &= 0 \\ \delta_{SL} \equiv \Delta_K \left[\gamma_s \cdot S (m_L^2 + \gamma_L^2) + \gamma_L \cdot L (m_s^2 + \gamma_s^2) \right] - 2\sqrt{\gamma_s \gamma_L} (\gamma_s \gamma_L + m_s m_L) \cdot R \\ + 2\sqrt{\gamma_s \gamma_L} (\gamma_L m_s - \gamma_s m_L) \cdot I &= 0 \end{aligned} \quad (4.6)$$

From the first two we easily get

$$R = \frac{\Delta_K}{2\sqrt{\gamma_s \gamma_L}} (\gamma_s \cdot S + \gamma_L \cdot L) \quad (4.7)$$

$$I = \frac{\Delta_K}{2\sqrt{\gamma_s \gamma_L}} \frac{\Delta m}{\gamma_L - \gamma_s} (\gamma_s \cdot S - \gamma_L \cdot L) \quad (4.8)$$

whereas the last condition in (4.6) needs a more detailed treatment. The reason why we did not neglect till now terms of order Γ_x/m_x is now apparent. Namely, in zeroth order of Γ_x/m_x we obtain

$$\delta_{SL}|_{S=L=1} = 0 \quad (4.9)$$

Hence to estimate how badly δ_{SL} deviates from zero it is necessary to include the next order of Γ_x/m_x . In this order using (4.5) δ_{SL} reads

$$\begin{aligned} \delta_{SL} &= \frac{\Delta_K}{\pi} \left(\frac{\gamma_L}{m_L} \right) \frac{\gamma_s}{\gamma_s - \gamma_L} \left[(\gamma_L - \gamma_s)^2 + \Delta m^2 \right] \left[(\gamma_s - \gamma_L) - \gamma_s \frac{\Delta m}{m_L} \right] \\ &\sim \frac{\Delta_K}{\pi} \Delta m^3 \left(\frac{\gamma_L}{m_L} \right) \end{aligned} \quad (4.10)$$

For the order of magnitude estimate in (4.10) we have used $\Gamma_s/\Delta m \sim \mathcal{O}(1)$. Strictly speaking this amounts to saying that the ansatz (4.1) is not consistent. Note, however, the following. The smallest mass scale parameter which appears in calculations involving the $K^0 - \bar{K}^0$ system is Δm . δ_{SL} in (4.6) has the canonical dimension 3. What eq.(4.10) then tells us is that as compared to the third power of the smallest mass scale δ_{SL} is zero, up to corrections of order Γ_x/m_x . Therefore to this accuracy everything is consistent so far. Clearly, by assuming $\Delta_K = 0$ we obtain $R = I = 0$.

The reader will have noticed that in making estimates like in eq.(4.10) we are relying on measured parameters of the $K^0 - \bar{K}^0$ system. In order not to lose track of the main point we will not examine simultaneously the systems $B^0 - \bar{B}^0$ and $D^0 - \bar{D}^0$. There the smallest mass scale parameter is not Δm but the corresponding difference in the widths $\Delta\Gamma$. The investigation of the consistency of (4.1) will be then slightly different in those systems. The general (hypothetical) case as well as cases of physical interest other than the $K^0 - \bar{K}^0$ system will be treated elsewhere [50].

Using only Khalfin's eq.(3.10) and the normalization condition (3.17) we have already pinned down the S , L , R and I in terms of known quantities like widths, masses and Δ_K . The equations (3.10) and (3.17)-(3.19) represent therefore an over-determined system. In contrast to situations discussed at the end of this section this is equivalent to a consistency check.

On account of the validity of eq.(3.10), proved for terms up to Γ_x/m_x , eq.(3.19) is bound to hold. We are therefore left with one more condition, namely (3.18). We will discuss the calculation in connection with (3.18) in some more detail since part of the steps will enter also the formulae of time evolution in section 5. The calculation will become more transparent by writing down explicitly the product $\sum_\beta \rho_{S,\beta}^*(m) \rho_{L,\beta}(m)$ with the spectral functions given by (4.1).

$$\sum_\beta \rho_{S,\beta}^*(m) \rho_{L,\beta}(m)|_{BW} = \frac{\sqrt{\gamma_S \gamma_L}}{\pi [(m - m_L)^2 + \gamma_S^2] [(m - m_L)^2 + \gamma_L^2]} \cdot \left\{ (a_R m^2 + b_R m + c_R) + i(a_I m^2 + b_I m + c_I) \right\} \quad (4.11)$$

with

$$\begin{aligned} a_I &= I, \quad b_I = (\gamma_S - \gamma_L) \cdot R - (m_S + m_L) \cdot I \\ c_I &= (\gamma_L m_S - \gamma_S m_L) \cdot R + (m_L m_S + \gamma_S \gamma_L) \cdot I \end{aligned} \quad (4.12)$$

and similar expressions for a_R , b_R and c_R . Next an ansatz for the partial fraction decomposition

$$\frac{a_I m^2 + b_I m + c_I}{[(m - m_L)^2 + \gamma_S^2] [(m - m_L)^2 + \gamma_L^2]} = \frac{C_I m + D_I}{(m - m_S)^2 + \gamma_S^2} + \frac{E_I m + F_I}{(m - m_L)^2 + \gamma_L^2} \quad (4.13)$$

leads as usually to a linear system for coefficients C_I , D_I , E_I and F_I

$$E_I = -C_I$$

$$\begin{aligned}
C_I \Delta m + D'_I + F'_I &= a_I \\
C_I \left[(m_L^2 + \gamma_L^2) - (m_s^2 + \gamma_s^2) \right] - 2D'_I m_L - 2F'_I m_s &= b_I \\
D'_I (m_L^2 + \gamma_L^2) + F'_I (m_s^2 + \gamma_s^2) + C_I \left[m_L (m_s^2 + \gamma_s^2) - m_s (m_L^2 + \gamma_L^2) \right] &= c_I
\end{aligned} \tag{4.14}$$

where we have used the redefinitions

$$D'_I \equiv D_I + C_I m_s, \quad F'_I \equiv F_I - C_I m_L \tag{4.15}$$

This system plays a double role in our discussion. It appears here as a middle step in the consistency check and is a necessary ingredient in the calculation of the time dependent transition amplitudes in the next section. Hence we feel that it is of enough importance to give the explicit solution of this system in appendix A. To perform the integral in (3.18) we need also

$$\begin{aligned}
\Lambda(R, I) &\equiv \int_0^\infty dm \frac{a_I m^2 + b_I m + c_I}{[(m - m_L)^2 + \gamma_s^2][(m - m_L)^2 + \gamma_L^2]} = \\
&- C_I \frac{\Delta m}{m_L} + \frac{D_I + C_I m_s}{\gamma_s} \left(\pi - \frac{\gamma_s}{m_s} \right) + \frac{F_I - C_I m_L}{\gamma_L} \left(\pi - \frac{\gamma_L}{m_L} \right) \\
&\quad + \mathcal{O}((\Gamma_x/m_x)^2) + \mathcal{O}((\Delta m/m_L)^2)
\end{aligned} \tag{4.16}$$

such that the condition (3.18) reduces to

$$\Lambda(R, I) = 0 \tag{4.17}$$

Taking the solutions for C_I , D'_I and F'_I in terms of R and I (see appendix A) and inserting them into (4.17) a lengthy calculation yields

$$\begin{aligned}
&R \cdot \Delta m \left[\Delta m^2 + (\gamma_s - \gamma_L)^2 \right] \left[2\pi + \frac{\gamma_s + \gamma_L}{m_L} \right] \\
&+ I \cdot (\gamma_s + \gamma_L) \left[\Delta m^2 + (\gamma_s - \gamma_L)^2 \right] \left[2\pi - \frac{\Delta m}{m_L} \frac{\Delta m}{\gamma_s + \gamma_L} \right] = 0
\end{aligned} \tag{4.18}$$

In performing this calculation it is not advisable to make too strong approximations right from the beginning. This is due to some cancellations which can occur. It is now trivial to compare eq.(4.18) with (4.7) and (4.8). In a simplified form eq.(4.18) is

$$\frac{I}{R} \simeq -\frac{\Delta m}{\gamma_s + \gamma_L} + \mathcal{O}(\Gamma_x/m_x) + \mathcal{O}(\Delta m/m_L) \tag{4.19}$$

which agrees with (4.7) and (4.8) when taking the approximation $S = L \simeq 1$.[†] The obvious conclusion here is that the one-pole ansatz (4.1) indeed passes the consistency check which has been imposed on us by a set of equations in section 3. This

[†]Yet a different way of displaying the consistency of (3.18) is described in section 5 (see there eqs.(5.8)-(5.10)).

check revealed that (4.1) is valid up to terms of order $\mathcal{O}(\Gamma_x/m_x)$, $\mathcal{O}(\Delta m/m_L)$. We emphasize that this is not a trivial check. To see this let us investigate the situation where we put by hand $\Delta_K = 0$. In this case we would obtain an homogeneous linear system whose only solution is $R = I = 0$. No information on the accuracy of (4.1) would follow from this. On the other hand keeping $\Delta_K \neq 0$ but dropping Khalfin's eq.(3.10) from the analysis we would end up with four equations ((3.17)-(3.19) for the four unknowns S , L , R , and I . Again no conclusion on the accuracy could have been reached. This displays once again the different nature of the $K^0 - \bar{K}^0$ system and also the usefulness of (3.10). As far as the *size* of one possible correction term ($\sim \Gamma_x/m_x$) is concerned the alert reader might object that this has been known all along as corrections to the exponential decay law. This is only partly true. As we have tried to argue above the presence of CP-violation alters the picture completely as only in this case equations (3.10) and (3.17)-(3.19) are an overdetermined system. In this context we remark that: 1. a consistency check has to be performed in any case as (4.1) could have been inconsistent for totally different reasons and 2. it is probably safer not to rely on restrictions obtained in the framework of a CP-conserving theory. Corrections of the order $\mathcal{O}(\Gamma_x/m_x)$ are of course expected to the *exponential decay law*, but the result here is more general as it explicitly states that corrections to *oscillatory terms* in $P_{K^0 K^0}(t)$ etc. coming from exact (unknown) spectral functions $\rho_{S,\alpha}$ and $\rho_{L,\alpha}$ will be of the same order. Both these corrections are totally different in nature since corrections to $\exp(-\Gamma t)$ are associated with the small/large time behaviour of the amplitudes whereas corrections to oscillatory terms might also arise for intermediate time scales. Indeed Khalfin's result on vacuum regeneration of K_S and K_L discussed in section 3 induces corrections of the latter type (see section 5). The nature of such corrections stemming from beyond (4.1) cannot be then apriori known and an analysis is required. That this analysis revealed $\mathcal{O}(\Gamma_x/m_x)$ and $\mathcal{O}(\Delta m/m_L)$ as limits of applicability of (4.1) means also that we can trust terms of order $\mathcal{O}(\Gamma_L/\Delta m)$, should such terms indeed appear along the line of further calculations. From now on we use

$$S = L \simeq 1 \quad (4.20)$$

unless otherwise stated.

We close this section by observing that the sum of (4.7) and (4.8) with $S = L \simeq 1$ is nothing else but the well known Bell-Steinberger unitarity relation [51], namely

$$\Delta_K (\gamma_S + \gamma_L - i\Delta m) = 2\sqrt{\gamma_S \gamma_L} \sum_{\beta} A_{S,\beta}^* A_{L,\beta} \quad (4.21)$$

The reason it appears here in a slightly different form (compare e.g. with [52]) is the different normalization of the amplitudes. Recently corrections to (4.21) of the order $\mathcal{O}(\Delta m/m_L)$ have been calculated (see the second reference in [45]). As shown above such corrections are indeed expected. Finally we note that for the analysis in this section it is immaterial whether or not $P_{K_L K_S}(t)$ is zero.

5 Time Development

Having convinced ourselves that the one-pole approximation (4.1) is consistent up to terms of order $\mathcal{O}(\Gamma_x/m_x)$ and $\mathcal{O}(\Delta m/m_L)$ we can proceed to calculate the matrix elements (3.11)-(3.13). With equations (4.7), (4.8) and (4.20) we have all necessary information to do so. We mentioned in section 3 that the ground state in $\text{Spec}(H)$ induces corrections to the exponential decay law (3.5). Since this also implies the integration domain $(0, \infty)$ in (3.11)-(3.13) we should handle such terms with care and make sure that all ‘new’ terms induced by the lower integration limit are indeed of strictly non-oscillatory type in (3.11)-(3.13). This is also important as we want to find out if Khalfin’s effect is correlated with small/large time scales. The relevant integrals have been calculated analytically in appendix B. We can infer from the expressions in appendix B that such terms contain the exponential integral function Ei [53]. We can safely neglect the terms with Ei as it should be clear that the simple ansatz (4.1) cannot account for the correctness of such non-oscillatory terms.

Let us now have a closer look at (3.11). In the one-pole approximation (4.1) $P_{K^0 K^0}(t)$ can be conveniently written as (see also (B.9) in appendix B)

$$P_{K^0 K^0}(t) = P_{\bar{K}^0 \bar{K}^0}(t) = \frac{1}{2\pi} \left\{ e^{-im_s t} \left(- \int_0^{-m_s/\gamma_s} dy \frac{e^{-i\gamma_s t y}}{y^2 + 1} + \int_0^\infty dy \frac{e^{-i\gamma_s t y}}{y^2 + 1} \right) + [S \rightarrow L] \right\} \quad (5.1)$$

We see that we have to calculate integrals of the following type

$$K^{(n)}(a) \equiv \int_0^\infty dx \frac{x^n}{x^2 + 1} e^{-iax} \\ J^{(n)}(a, \eta) \equiv \int_0^\eta dx \frac{x^n}{x^2 + 1} e^{-iax} \quad (5.2)$$

Collecting only oscillatory terms from the integrals in appendix B we obtain the same expression as in WW-approximation (this of course is not a surprise recalling that our concern here is the last equation in (2.11) where only $P_{K^0 \bar{K}^0}(t)$ and $P_{\bar{K}^0 K^0}(t)$ play a role)

$$P_{K^0 K^0}(t) = P_{\bar{K}^0 \bar{K}^0}(t) = \frac{1}{2} \left\{ e^{-im_s t} e^{-\gamma_s t} + e^{-im_L t} e^{-\gamma_L t} \right\} + N_{K^0 K^0}(t) \quad (5.3)$$

where $N_{K^0 K^0}(t)$ denotes all non-oscillatory terms present in the integral. $N_{K^0 K^0}(t)$ can, in principle, be extracted from equations (B.1)-(B.5) but as we said before we cannot trust such terms to be the correct non-oscillatory corrections.

One more comment is order. Putting γ_s/m_s to zero the sum of the two integrals in (5.1) can be compactly written as

$$\int_{-\infty}^\infty dy \frac{e^{-i\mu y}}{a^2 + y^2} = \frac{\pi}{a} e^{-\mu a} \quad (5.4)$$

which of course means that

$$N_{K^0\bar{K}^0}(t) \rightarrow 0 \text{ as } \frac{\Gamma_{S/L}}{m_{S/L}} \rightarrow 0 \quad (5.5)$$

in agreement with what we said at the beginning of section 3 (see discussion below eq.(3.5)).

Similarly the integration in (3.12) and (3.13) can be done analytically (see (B.10) in appendix B) and the result reads

$$\begin{aligned} P_{K^0\bar{K}^0}(t) &= \frac{1}{4p^*q} \left\{ e^{-im_S t} e^{-\gamma_S t} [1 + \kappa_S] - e^{-im_L t} e^{-\gamma_L t} [1 + \kappa_L] \right\} + N_{K^0\bar{K}^0}(t) \\ P_{\bar{K}^0 K^0}(t) &= \frac{1}{4pq^*} \left\{ e^{-im_S t} e^{-\gamma_S t} [1 - \kappa_S] - e^{-im_L t} e^{-\gamma_L t} [1 - \kappa_L] \right\} + N_{\bar{K}^0 K^0}(t) \end{aligned} \quad (5.6)$$

where $N_{K^0\bar{K}^0}(t)$ and $N_{\bar{K}^0 K^0}(t)$ are again non-oscillatory terms containing the exponential integral function Ei and $\kappa_{S/L}$ are given by

$$\begin{aligned} \kappa_S &= -2i \frac{\sqrt{\gamma_S \gamma_L}}{\gamma_S} [D'_I - i\gamma_S C_I] \\ \kappa_L &= +2i \frac{\sqrt{\gamma_S \gamma_L}}{\gamma_L} [F'_I + i\gamma_L C_I] \end{aligned} \quad (5.7)$$

The parameter C_I , D'_I and F'_I are defined as solutions of the linear system (4.14). Equation (5.6) together with (2.11) shows that Khalfin's effect depends crucially on the size of the quantities $\kappa_{S/L}$. We could, in principle, calculate these quantities taking the solutions C_I , D'_I and F'_I from appendix A. There is, however, a more elegant way by going back to (4.14). This linear system fixes C_I , D'_I and F'_I in terms of R and I (eqs.(4.7)-(4.8)) the latter being kept at this stage in section 4 arbitrary i.e. in any order of Γ_x/m_x . But we know now that we are allowed to keep only the zeroth order of Γ_x/m_x . Then R , I taken together with (4.20) and a redefinition of the form

$$\begin{pmatrix} \tilde{C}_I \\ \tilde{D}_I \\ \tilde{F}_I \end{pmatrix} = 2 \frac{\sqrt{\gamma_S \gamma_L}}{\Delta_K} \begin{pmatrix} C_I \\ D'_I \\ F'_I \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} \tilde{a}_I \\ \tilde{b}_I \\ \tilde{c}_I \end{pmatrix} = 2 \frac{\sqrt{\gamma_S \gamma_L}}{\Delta_K} \begin{pmatrix} a_I \\ b_I \\ c_I \end{pmatrix} \quad (5.8)$$

convert (4.14) into a homogeneous linear system in the limit $\Gamma_x/m_x \rightarrow 0$

$$\begin{pmatrix} -\tilde{a}_I & 1 & 1 \\ -\tilde{b}_I & -2m_L & -2m_S \\ -\tilde{c}_I & m_L^2 + \gamma_L^2 & m_S^2 + \gamma_S^2 \end{pmatrix} \begin{pmatrix} \tilde{C}_I \\ \tilde{D}_I \\ \tilde{F}_I \end{pmatrix} = 0 \quad (5.9)$$

Since the determinant [‡] of the coefficient matrix in (5.9) is non-zero we get only a trivial solution

$$\tilde{C}_I = \tilde{D}_I = \tilde{F}_I = 0 \quad (5.10)$$

[‡]Demanding the determinant to be zero gives $\tilde{b}_I - \tilde{a}_I \tilde{c}_I = 0$ and in the suitable accuracy $\gamma_S^2 - \gamma_L^2 \simeq 0$ or $(\gamma_S^2 - \gamma_L^2)/\Delta m^2 \simeq -2(m_S + m_L)/\Delta m$. Clearly both these relations are only hypothetical and not valid in the $K^0 - \bar{K}^0$ system.

This immediately implies that

$$\kappa_S = \kappa_L = \Delta_K + \mathcal{O}(\Gamma_X/m_X) + \mathcal{O}(\Delta m/m_L) \quad (5.11)$$

Equipped with this simple result eq.(5.6) take the familar form

$$\begin{aligned} P_{K^0\bar{K}^0}(t) &= \frac{p}{2q} \left\{ e^{-im_S t} e^{-\gamma_S t} - e^{-im_L t} e^{-\gamma_L t} \right\} + \text{non - osc. terms} \\ P_{\bar{K}^0 K^0}(t) &= \frac{q}{2p} \left\{ e^{-im_S t} e^{-\gamma_S t} - e^{-im_L t} e^{-\gamma_L t} \right\} + \text{non - osc. terms} \end{aligned} \quad (5.12)$$

Up to non-oscillatory terms these equations are equivalent to the WW-expressions. What we have shown is that indeed corrections to oscillatory terms due to Khalfin's general result will appear in (5.12), but they are necessarily of the order $\mathcal{O}(\Gamma_X/m_X)$, $\mathcal{O}(\Delta m/m_L)$. This follows from the fact that the one-pole approximation is trustable only up to such terms. In the calculation with the one-pole ansatz any term whose order of magnitude is much bigger than $\mathcal{O}(\Gamma_X/m_X)$, $\mathcal{O}(\Delta m/m_L)$, like $\Gamma_L/\Delta m$, would be then still acceptable. But such a term does not show up along the line of the calculation. It should also be appreciated that such corrections have nothing to do with small/large time behaviour of the transition amplitudes (i.e. they are not interrelated to the usual corrections to the exponential decay law). This is evident from the way $\kappa_{S/L}$ enters (5.6).

Finally the answer to the question we have put forward in the form of equation (2.13) can also be given by a simple equation, namely

$$P_{K_L K_S}(t) = -P_{K_S K_L}(t) = 0 + \mathcal{O}(\Gamma_X/m_X) + \mathcal{O}(\Delta m/m_L) \quad (5.13)$$

Had we not Khalfin's theorem discussed in section 3, it would be completely legitimate to assume $P_{K_L K_S}(t)$ to be strictly zero. Our result agrees with the conclusion of ref. [46] reached there in a different way. We postpone any further discussion to the next section where we will give a summary. In the end we compare our result with expressions obtained by Khalfin who arrives at a equation similar to (5.6) [6]. To obtain his results we have to make only the following replacement in eq.(5.6)

$$\kappa_S \rightarrow \frac{-2i\sqrt{\Gamma_S \Gamma_L}}{\Delta m + i(\Gamma_S + \Gamma_L)}, \quad \kappa_L \rightarrow \kappa_S^* \quad (5.14)$$

As explicitly shown in [46] the numerical value of κ_S would then be

$$\kappa_S \sim 0.06 e^{i\pi/4} \quad (5.15)$$

The effect would then indeed be of the order $\Gamma_L/\Delta m$ as can be seen from the equation

$$|P_{K^0\bar{K}^0}(t)|^2 \simeq \frac{1}{4} \left\{ e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\gamma_S + \gamma_L)t} \left[\cos(\Delta m t) - 0.4 \times 10^{-3} \sin(\Delta m t) \right] \right\} \quad (5.16)$$

We have, however, shown that this is an overestimate by several orders of magnitude. The difference between Khalfin's approach and ours is essentially our consistent treatment of the one-pole approximation in section 4.

6 Conclusions

It is satisfactory to arrive after lengthy calculations at familiar expressions of the Weisskopf-Wigner approximation. More so as our starting point was completely different from the WW-approach. This not only gives us more confidence in the WW-approximation whose equations, as we know, are of utmost importance for the $K^0 - \bar{K}^0$ system, but has also the virtue that one is able to derive the limitations of the WW-approximation for the oscillatory as well as for the exponential terms. We have emphasized that corrections to the oscillatory terms are different in nature from corrections arising from small/large time behaviour of the amplitudes. It turned out, however, that both such corrections must be of the order $\mathcal{O}(\Gamma_x/m_x)$, $\mathcal{O}(\Delta m/m_L)$. This is apriori not evident due to the specifics of the $K^0 - \bar{K}^0$ system where beside Γ_x/m_x quantities like $\Gamma_L/\Delta m$ do appear. The reanalysis of the present paper was also necessary in view of a claim of Khalfin that new effects in connection with the non-zero vacuum regeneration of K_S and K_L are of the order of $\Gamma_L/\Delta m$. Let us recapitulate the steps which have led to our result. We have presented two of Khalfin's theorems. One was eq.(3.10) which played a crucial role in our analysis. Actually without this equation no conclusion on the validity of the one-pole approximation could have been reached. The other one was the surprising result on the existence of K_S and K_L vacuum regeneration, an effect usually associated with interactions of K_S and K_L in matter. Although this result is quite 'exotic' the author of the present paper could not find a loop-hole in the arguments which led to this result. The vacuum regeneration of K_S and K_L goes against what one would intuitively expect and what one is normally used to. Note, however, the this 'intuition' is based on quantum mechanical systems where the unstable states have zero overlap. $|K_S\rangle$ and $|K_L\rangle$ have non-zero overlap, a singled-out property which is then responsible for counter-intuitive effects. The proof of Khalfin's result relies on well established formalisms of Quantum Mechanics (eqs.(3.1)-(3.3)) and seems therefore hard to dispute once we assume that $|K_S\rangle$ and $|K_L\rangle$ are given as in (2.1). To estimate the size of such an effect we had to perform a consistency check of the one-pole approximation (4.1). The outcome of this check provided us with limits of the applicability of (4.1) and the determination of apriori unknown variables (combinations of decay amplitudes). Indeed the difference between the present paper and the result obtained by Khalfin can be traced back to exactly this point. In a subsequent step we have derived the time evolution of the system starting from the equations (3.11)-(3.13). The formulae so obtained agreed with expressions from the WW-formalism. This in turn implied that the effect of vacuum regeneration of K_S and K_L is necessary small and of the order of $\mathcal{O}(\Gamma_x/m_x)$, $\mathcal{O}(\Delta m/m_L)$.

Our estimate does not render the general result of Khalfin useless as in fact the effect is non-zero. Furthermore we know from this result that on quite general

grounds

$$\frac{P_{K^0\bar{K}^0}(t)}{P_{\bar{K}^0K^0}(t)} \neq \text{const} \quad (6.1)$$

Any test therefore which as a starting assumption relies instead on (2.20) [37] should be then carefully reconsidered.

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Appendix A

We list here the solutions of the linear system (4.14). Since the expressions are lengthy it is convenient to use the following notational abbreviations

$$\begin{aligned} X_+ &= \gamma_s^2 + \gamma_L^2, \quad X_- = \gamma_s^2 - \gamma_L^2 \\ Z &= m_L \gamma_s^2 - m_s \gamma_L^2 \\ Y_I &= \Delta m^4 + 2\Delta m^2 X_+ + X_-^2 \end{aligned} \quad (\text{A.1})$$

The solutions in terms of a_I , b_I and c_I defined in eq.(4.12) then read

$$\begin{aligned} F'_I \cdot Y_I &= a_I \left[\Delta m^2 m_L^2 - (m_L^2 + \gamma_L^2) X_- - \Delta m m_L X_+ + (m_L + m_s) Z \right] + \\ &\quad b_I \left[\Delta m^2 m_L + Z - \Delta m \gamma_L \right] + \\ &\quad c_I \left[\Delta m^2 + X_- \right] \end{aligned} \quad (\text{A.2})$$

$$\begin{aligned} D'_I \cdot Y_I &= a_I \left[\Delta m^2 m_s^2 + (m_s^2 + \gamma_s^2) X_- + \Delta m m_s X_+ - (m_L + m_s) Z \right] + \\ &\quad b_I \left[\Delta m^2 m_s - Z + \Delta m \gamma_s \right] + \\ &\quad c_I \left[\Delta m^2 - X_- \right] \end{aligned} \quad (\text{A.3})$$

$$\begin{aligned} C_I \cdot Y_I &= a_I \left[\Delta m^3 - \Delta m (m_s^2 + m_L^2 - \gamma_s^2 - \gamma_L^2) - (m_s + m_L) X_- \right] - \\ &\quad b_I \left[\Delta m (m_s + m_L) + X_- \right] - \\ &\quad c_I 2\Delta m \end{aligned} \quad (\text{A.4})$$

Appendix B

This appendix contains the relevant integrals appearing in (3.11)-(3.13) with $\rho_{s,\alpha}$ and $\rho_{L,\alpha}$ approximated by (4.1). The integrals $K^{(n)}(a)$ and $J^{(n)}(a, \eta)$ are defined in (5.2). We have [53]

$$K^{(0)}(a) = \int_0^\infty dx \frac{1}{x^2 + 1} e^{-iax} = \frac{\pi}{2} e^{-a} - \frac{i}{2} \left[e^{-a} Ei(a) - e^a Ei(-a) \right] \quad (\text{B.1})$$

where Ei are transzendent functions called exponential integral functions. Any other integral $K^{(n)}$ with $n > 0$ can be obtained from (B.1) by differentiating (B.1) with respect to the variable a . For instance

$$K^{(1)}(a) = \int_0^\infty dx \frac{x}{x^2 + 1} e^{-iax} = -i \frac{\pi}{2} e^{-a} - \frac{1}{2} \left[e^{-a} Ei(a) + e^a Ei(-a) \right] \quad (\text{B.2})$$

The integral $J^{(n)}$ are more complicated. To obtain $J^{(0)}$ we have used the Fourier identity

$$\int_0^\eta f(x)dx = \frac{1}{2\pi} \int_{-\infty}^\infty dy \frac{e^{iny} - 1}{iy} \int_{-\infty}^\infty e^{-iy\xi} f(\xi) d\xi \quad (\text{B.3})$$

Here we quote only the result

$$J^{(0)}(a, \eta) = \int_0^\eta dx \frac{1}{x^2 + 1} e^{-iax} = -\frac{1}{2i} \left\{ -i \operatorname{sgn}(\eta) e^{-a} + e^{-a} Ei(a(1 - i\eta)) - e^a Ei(-a(1 + i\eta)) - e^{-a} Ei(a) + e^a Ei(-a) \right\} \quad (\text{B.4})$$

where $\operatorname{sgn}(\eta)$ is the sign of η . Again $J^{(n)}$, $n > 0$ can be obtained from (B.4) by differentiation of (B.4) with respect to a

$$J^{(1)}(a, \eta) = \int_0^\eta dx \frac{x}{x^2 + 1} e^{-iax} = -\frac{1}{2} \left\{ i \operatorname{sgn}(\eta) e^{-a} - e^{-a} Ei(a(1 - i\eta)) - e^a Ei(-a(1 + i\eta)) + e^a Ei(-a) + e^{-a} Ei(a) \right\} \quad (\text{B.5})$$

The reason why we have to distinguish between the signs of η has to do with the following property of the exponential integral function Ei [53]

$$Ei(x \mp i0) = Ei(x) \pm i\pi, \quad x > 0 \quad (\text{B.6})$$

One can check (B.4) by using the integral representation

$$Ei(\pm xy) = \pm e^{\pm xy} \int_0^\infty dt \frac{e^{-xt}}{y \mp t}, \quad \Re(y) > 0, x > 0 \quad (\text{B.7})$$

and differentiating both sides of (B.4) with respect to η . We also mention here the connection of $Ei(x)$ with the incomplete beta function $\Gamma(\alpha, x)$ [53] through

$$\Gamma(0, x) = -Ei(-x) \quad (\text{B.8})$$

Finally the integrals (B.1)-(B.5) enter (3.11)-(3.13) through the expressions

$$\int_0^\infty dm \sum_\alpha |\rho_{s,\alpha}(m)|^2 e^{-imt} = \frac{1}{\pi} e^{-im_s t} \left[-J^{(0)}(\gamma_s t, -m_s/\gamma_s) + K^{(0)}(\gamma_s t) \right] \quad (\text{B.9})$$

and

$$\begin{aligned} & \int_0^\infty dm \sum_\beta \Im m \left(\rho_{s,\beta}(m) \rho_{L,\beta}^*(m) \right) e^{-imt} = \\ & - \frac{\sqrt{\gamma_s \gamma_L}}{\pi} \int_0^\infty dm \frac{a_I m^2 + b_I m + c_I}{[(m - m_L)^2 + \gamma_s^2] [(m - m_L)^2 + \gamma_L^2]} e^{-imt} = \end{aligned}$$

$$\begin{aligned}
& - \frac{\sqrt{\gamma_s \gamma_L}}{\pi} \left\{ \frac{e^{-im_s}}{\gamma_s} \left[D'_I \cdot \left(-J^{(0)}(\gamma_s t, -m_s/\gamma_s) + K^{(0)}(\gamma_s t) \right) \right. \right. \\
& \quad \left. \left. + \gamma_s C_I \cdot \left(-J^{(1)}(\gamma_s t, -m_s/\gamma_s) + K^{(1)}(\gamma_s t) \right) \right] \right. \\
& \quad + \frac{e^{-im_L}}{\gamma_L} \left[F'_I \cdot \left(-J^{(0)}(\gamma_L t, -m_L/\gamma_L) + K^{(0)}(\gamma_L t) \right) \right. \\
& \quad \left. \left. - \gamma_L C_I \cdot \left(-J^{(1)}(\gamma_L t, -m_L/\gamma_L) + K^{(1)}(\gamma_L t) \right) \right] \right\} \tag{B.10}
\end{aligned}$$

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